Sharing the Beauty of Mathematics Through Making and Design Thinking

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Background

In this chapter I introduce *Mathematical Design*, a course developed to bring the beauty and creativity of mathematics to general education students at Queens College. In this project-based course, students learn about mathematical functions as tools to design works of art on the online graphing calculator Desmos. They then translate this digital artwork onto paper through the use of an AxiDraw[1] pen plotter. The course is accessible and enjoyable for college students of any background and for many it provides their first time experiencing mathematics in a positive way.

Queens College is a member institution of the City University of New York, the largest urban university system in the United States. Queens College is predominantly a commuter campus with approximately 15,000 students whose diversity reflects the population of its location, with students who represent over 150 countries and speak over 90 languages. Queens College is the region's primary engine of economic mobility with 35% of undergraduate students being the first in their family to go to college.

Students at Queens College receive a well-rounded liberal arts education which is scaffolded through the general education framework known as Pathways. In this framework students must take courses that address different aspects of a liberal arts education including: writing; life and physical sciences; world cultures; individual and society; and mathematics and quantitative reasoning (MQR). General education courses are intended to be taken by students in their first years or two of college without needing specialized knowledge. *Mathematical Design* was created to satisfy the MQR requirement for students who are majoring in the creative arts but is open to students from any discipline.

The students who enroll in this course are often afraid of mathematics. To them, mathematics has always been synonymous with having to "get the right answer". When a student feels unsupported in this goal of finding the right answer and repeatedly gets negative reinforcement from their teachers in the form of bad grades, it is natural that this turns into a fear of failing (to get the right answer) and a general feeling of math anxiety. In my four years of teaching *Mathematical Design*, I often encounter students who are anxious about even starting to approach a problem.

Let us contrast this with how many students experience courses in art and creative expression. Experimentation and play are valued and supported by instructors. Even in language arts classes, students are exposed to advanced concepts without an expectation that they need to replicate that level of work. Elisabeth Heathfield and George Hart in their *Making Math Visible* project write:

> "Just as exposure to great books can entice students to learn to read, beautiful mathematical objects may fuel students' desire to investigate topics in mathematics and give them an opportunity to acquire a positive lifetime perspective towards math. Traditional classroom walls are often overflowing with language-based displays of learning, while the beauty of math is rarely seen." (Hart and Heathfield, 2017)

Taking a cue from these more positive experiences in other subjects, students in *Mathematical Design* are given the opportunity to experiment and play with mathematical functions and see creative illustrations of mathematical concepts. In doing so, students explore and appreciate the beauty of mathematics through a hands-on experience.

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[1] The pen plotter we used was an AxiDraw machine, which is no longer manufactured. Its technology has been integrated into the Bantam Tools NextDraw machine after they acquired AxiDraw's manufacturer.

In the next sections, I show how students go from being afraid of math to making mathematical art in one semester's time.

Course Description and Student Deliverables

Mathematical Design is a course in which students build mathematical knowledge and technical skills, harness their creativity through making and design thinking, and develop their writing and communication skills. This section presents an overview of the course structure; I discuss the details of the content later.

The basic principle is that students digitally create artistic scenes involving the curves of mathematical functions. *Mathematical Design* incorporates active learning through a project-based approach. The structure of the course builds on my past experience in teaching project-based classes with active learning techniques such as *Mathematical Computing*, *Mathematical Modeling*, and *Combinatorics*. I have previously written about the implementation of the *Mathematical Computing* course. (Hanusa, 2023)

The students physically realize their mathematical artwork on paper using machines in the Queens College Makerspace. As the semester progresses, students grow their confidence, mathematical knowledge, and technical abilities on the machines, so they are able to create more and more elaborate pieces. Figure 1 is a visualization of the workflow the students use. Students learn and use the online calculator [Desmos] to design their artwork. They then transfer their artwork to the free open-source image editing software Inkscape. Students then use an extension in [Inkscape] to control the AxiDraw machines to move its arms along its *x*- and *y*-axes to replicate the scene with a writing instrument of their choice on a medium of their choice.

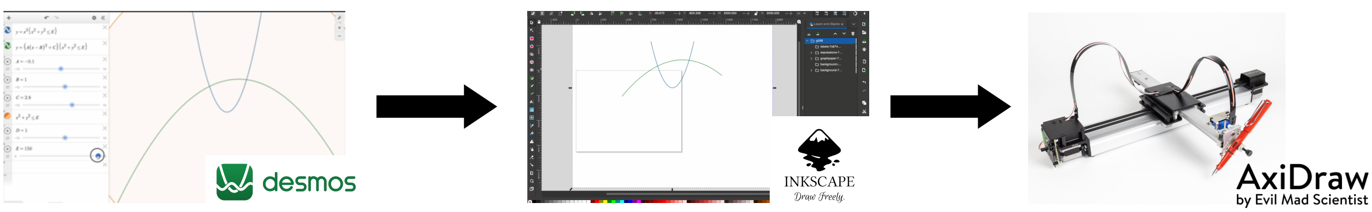

Figure 1. The workflow students use to create mathematical art. Students create digital artwork in Desmos, import it into Inkscape, and use the AxiDraw machine to draw the artwork.

The course centers around a series of three student-driven projects and a portfolio. The first project is to create a simple sketch relying on the mathematical concept of transformations. The goal is for students to become comfortable with the workflow and to develop basic skills in mathematics, coding with Desmos and with the AxiDraw machine. The second project asks students to push themselves further. They will have learned more skills in mathematics, in Desmos, in Inkscape, and are asked to use more advanced materials on the AxiDraw. The third project asks students to branch out to realize their mathematical art on another machine in the Makerspace, such as the 3D printer, the laser cutter, or the embroidery machine. (See Figure 2.) They learn mathematics that most have never seen before and get a chance to explore the new machines, which requires learning new workflows.

Each of these three projects requires submitting three deliverables: a piece of art, the code used to create the art, and a written lab report detailing their process and the choices they made along the way. On the day that the artwork is submitted we perform a gallery walk as a class, where all the artwork is displayed in one space and each student gets a chance to talk about the work they have done in a low-stakes environment.

Figure 2. Students using the 3D printers and the embroidery machines in the Makerspace.

The portfolio project asks students to document and reflect on their evolution over the semester---they assemble final pieces of work, along with inspirations, explorations they completed, draft pieces of work and write a reflection in which students detail their progress as a mathematician, an artist, and a maker over the course of the semester.

The project descriptions and timelines, discussed further below, are written in such a way to scaffold the students' progress but not constrain the subject of their artwork. In this way, students have the agency to create their artwork the way they want and it becomes a conduit that engages a student's lived experiences. The sharing of student progress through groupwork, art walks, or presentations allows students to learn from each other's experience and learning process to bootstrap the understanding throughout the class. This allows us to harness the diversity of the students in the classroom and makes the course content accessible for students at all levels of understanding.

Setting the stage for a successful semester

I have found that the best way to ensure a successful experience for the students is to instill from the beginning of the semester a supportive group environment in which students feel comfortable volunteering suggestions without feeling judged for not knowing something or putting forth an idea that is not good enough. This is an environment that recognizes and values productive failure and the voice of everyone in the room.

Therefore, the course must be constructed to ensure a student voice throughout the semester. It is crucial on the first day to model expectations for the rest of the semester. As most students are commuters and come from many different majors and disciplines, students in the course have limited opportunities to interact outside of class time. For this reason, I start the semester with introductions, asking students to share a little bit about themselves, which can include their majors, hobbies, creative outlets, and employment. This allows students a chance to see that their voice is valued and to perceive other students in the class who have similar interests, and which might form the basis for future collaborations.

Especially for the population in this course and on the first day of class, I find it important to acknowledge the elephant in the room by asking students to do some word association by sharing the words that come to mind when they hear the word "math". Invariably, a few students start by naming mathematical concepts. Then either a brave student volunteers (or I gently prod) about their feelings, and this unleashes ripples of sentiments surrounding math anxiety from throughout the classroom. I have found that by letting students know that I am aware of the ways in which many students perceive mathematics, and by speaking to their concerns, this allows for students to let down their guard and be more accepting of the mathematical concepts to come. As an instructor it is important to identify this fear, design the course with this in mind, and give students permission to engage fully with the material.

Additional experiences throughout the semester build the community intentionally. For example, I like to use in-class Desmos tutorial activities (aligned with the mathematical content below) that are completed as a group with a driver (someone working the keyboard) and active passengers (who provide feedback to the driver). It works best when the driver is not the strongest of the group – having to use one's words to explain what is or is not working means that everyone can benefit from the experience and the communal knowledge building.

Furthermore, multiple homework assignments near the beginning of the semester ask students to work together with one or more classmates for at least one hour on a task (such as creating a mathematical scene or practicing the AxiDraw workflow), and is paired with a metacognition exercise to write a paragraph (5-7 sentences) on a topic such as "What did you learn in a group that you would not have learned by completing the homework assignment yourself?". Student responses often reinforce that working together makes the learning deeper, broader, and more fun, and it makes it less likely that a student will give up when they hit a wall that they don't know how to traverse alone. (See Figure 3.)

Figure 3. Students using the AxiDraw machine in the Makerspace in Spring 2023 and Spring 2024.

These activities lead to a community where individuals can make progress in a supportive group environment and where students understand that they do not need to rely on their instructor as the only expert in the room. This comfortable and collaborative atmosphere gives students an opportunity to experience mathematics in a positive way, which for many of these students is a first.

Developing Mathematics and Computing Skills

Every mathematical idea in the course is presented so that it can be used for artistic experimentation and as a creative tool. Each mathematical image is a collection of graphs of functions, so the mathematical instruction begins with the definition of a function, a discussion of the difference between standard functions (of the form $y = f(x)$) and implicit functions (of the form $f(x, y) = c$), a dictionary of *parent function* building blocks, and the development of *function sense*: the different properties that one should expect from each parent function. Students have seen these topics before, but many never internalized them or forgot the details.

This introduction to functions is coupled with interactive activities in the online mathematical calculator Desmos. Desmos has a shallow learning curve, which allows students to jump right in and play with the functions using interactive features. After an introductory tutorial to the software in which students experience basic-to-intermediate functionality of Desmos, students are taught to initiate this functionality themselves through a sequence of tutorials. The most important tool at a beginner's disposal is the slider. Introducing the variables a, b, and c to convert an equation involving the parent function $y = x^2$ to the more general form $y = ax^2 + bx + c$, the static graph of the parabola becomes an interactive display in which students move sliders associated to a, b, and c to explore the entire range of possible parabolas. This is incredibly empowering because it changes an instructor-centered equation into a student-driven experience.

I teach students to use this technique to modify parent functions in Desmos and record the data that makes the curves they like by copying the precise values of the variables into a new cell. Students learn about *lists* in Desmos, which can be used to store the information from multiple curves in one compact form. Students learn the tool of *domain restriction*, which allows them to restrict a function that extends across the screen in an unbounded fashion into a *snippet* that has a desired size. The mathematical content that rounds out the first third of the course is the concept of a translation of a graph. By this point of the semester, students are able to create scenes involving multiple translated snippets, which is the expectation for their first project.

The second third of the course starts with dilations, reflections, and combining transformations. Then the universe of available functions expands further by discussing combining functions through addition, multiplication, and composition, and introducing trigonometric functions. The mathematical content here rounds out by discussing polar coordinates, functions, and curves. It is worth remembering that the focus of these topics is giving the students these new functions and transformations as tools to create more elaborate curves, not on the need to understand every aspect of them. *Fifty Famous Curves* (Kokoska, n.d.) is a visual dictionary that students can refer to for inspiration; they are encouraged to share equations, curves, and other inspiration on our online discussion board.

Students become more comfortable exploring the capabilities of Desmos, including applying function restrictions that are two-dimensional (such as $x^2 + y^2 < 1$) and including user-specified colors into their designs. They also are expected to do improve their ability to organize and document in their notebooks using text cells and folders.

The mathematical concepts in the last third of the course include parametric functions, parametric curves, converting standard and polar functions to parametric functions, rotating curves, and linear interpolations between curves. Advanced students might explore algorithmic techniques, adding randomness to create generative art, Mobius transformations, and complex variables. However, most of the effort in this third of the course goes into learning the workflow on the machine they will be using for their third project.

By the end of the course, my goal is that students are able to start with an idea, analyze the situation, and identify and define the mathematical foundations and computing requirements appropriate to its solution.

Using Design Thinking to Encourage Creativity in Mathematics

When a math instructor teaches students a mathematical concept that involves a multi-step process, they might present the theory, provide students with an algorithm to complete the steps, model the process, and ask students to practice those steps. Once students gain comfort with the algorithm, they are better equipped to solve problems that involve those techniques. However, having the knowledge of a mathematical tool is not the same as being able to use it to create mathematical art.

While most students in the class have harnessed their creative talents, they often have not considered the design process. The combination of the unfamiliar mathematical tools and the students' existing math anxiety often leads them to be overwhelmed and unsure how to be creative with mathematics. As a result, it has been imperative that this course spends time teaching theory of art and design to give the students the scaffolding and permission to be creative with mathematics. I have found the *double diamond design framework* to be a useful framework to teach students how to approach solving a vaguely defined problem. (See Figure 4.)

The first stage of the framework is discovery, in which one brainstorms the universe of possible directions to solve the problem, followed by the definition stage, in which one refines the possible directions to a well-defined problem. The third stage is development, where one once again opens up the set of possibilities to find the best way to approach the now-defined problem; this is followed by the delivery stage in which one hones the possible solutions to the optimal one.

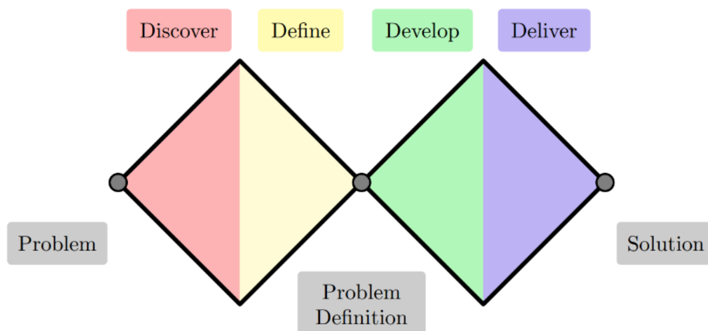

Figure 4. The Double Diamond Design Framework

In *Mathematical Design* projects, the vaguely defined problem is to make some mathematical art that satisfies certain parameters. We talk through how to apply each of these phases, and their homework assignments that lead up to their first project scaffold progression through the entire design framework.

In the discovery stage, students are asked to come up with three scenes involving different mathematical concepts. For example, they might come up with one scene involving lines, one scene involving circles, and one scene involving parabolas. In the defining stage, the student analyzes the three different scenes and finds different properties that they appreciate about each of them. They then need to choose one of the three scenes that they want to develop further, for example the scene with circles. This then helps to fully define the problem to be: What is the best way to make a scene involving circles?

Then there is another exploration stage in which the student creates three new scenes, each only involving circles, perhaps drawing inspiration from the aspects of the discarded scenes in the first diamond. Once the student has explored the universe of possible scenes involving only circles, the delivery phase begins by choosing one of the circle scenes and honing it until there is a final piece of art, solving the original problem.

One might ask whether following a prescribed framework should be considered creativity and whether the result of the process should be considered a work of art. I would argue that they should. We spend time during our class discussing what makes an object a piece of art. Much of the discussion centers around the idea of *intentionality*. What was the goal of the artist? What were they trying to do, accomplish, or convey with their piece? Each of the steps in the design process, while structural, are not prescriptive, and will be approached by different students in different ways.

The class also spends time discussing the elements of art and the principles of design (J. Paul Getty Museum, 2011) to give the students the language they need to explore the universe of options and hone their artwork intentionally. Elements of art involve ideas of line, shape, space, color, and texture, while principles of design include concepts of balance, emphasis, repetition, rhythm, and unity. I make sure to emphasize the exploratory part of each of the diamonds so that students understand that creativity is not a linear process. Students write about their process and the choices they make in their lab report.

Developing a Maker Mindset

I aim to blend the structure of the double diamond design framework with the freedom and self-determination of the maker movement. Students are guided to develop a *maker mindset* in which asking questions, tinkering, taking risks, and productive failure are all encouraged and expected. This starts with an orientation into the Queens College Makerspace where students are introduced to the space and the available tools; once they have completed the tour they have access to the space whenever they want.

The workflow to realize a piece of art on a pen plotter has many steps and many places of failure. Students must design their artwork in Desmos, modify the settings of their scene to remove the axes and grid, export their artwork from Desmos to an svg file, import this file into Inkscape, resize the imported shapes to fit the canvas, remove invisible layers, choose pens and paper, prepare and align the AxiDraw machine, and use the Axidraw extension menu in Inkscape to initialize the print.

This workflow is difficult for beginners. Their first homework assignment involving plotting is to print something that is as simple as possible, so that the design of the artwork is not the barrier to having the process succeed. I walk the students through this workflow once in class, then I ask them to work together outside of class to practice because each student understands different steps with different levels of proficiency, and working together means that they can support each other. This builds solidarity, encourages the maker mindset, and gives another set of eyes to resolve issues or misconceptions with their code and the workflow. It quickly reinforces that theory is different from practice and that one will succeed only by trying, making mistakes, and overcoming them. This delegation of responsibility is also more feasible for classroom management because peer help requires less instructor involvement than an instructor helping each student individually.

It is important to coordinate between the instructor and the makerspace staff to ensure that the makerspace staff is not overwhelmed by the influx of students and that the students understand the community expectations of student ownership of the process. The flow of students through the space worked best when a reservation system had been implemented so that students could reserve an AxiDraw machine in advance and students learned to not expect to be able to use the machine at the last minute before an assignment was due. It is important that users have 60-90 minute blocks of uninterrupted time for each assignment. This takes into account the start up time, unexpected hiccups, and time for prototyping and finalizing one's art.

Key Skills: Documenting and Communicating

An additional explicit goal of *Mathematical Design* is to encourage students to build their competencies in oral and written communication. They are expected to engage genuinely with classmates and the material by participating in class, discussing and solving problems in a group setting, and practicing giving and receiving constructive feedback. They are also expected to effectively communicate the decisions made throughout the intentional creation of their artwork.

I introduce a habit of documentation through carefully planned homework assignments to encourage the students to make feasible and incremental progress over time. There is a small homework assignment after every class that asks students to internalize the skills learned during class, produce something that showcases the skill, and share it with the class on the online discussion board. I ask students to share the result of the work as an image file and share a link to the online Desmos notebook so that other students can see how they approached the assignment. These assignments may also include sharing inspirational artwork or a picture of themselves or their artwork in process. These check-ins become invaluable when they need to write about their projects or compile documentation for their portfolio.

This homework and documentation structure is also engineered to build community in the class and make peer-created materials accessible for students who are feeling behind and are uncomfortable asking for help. I can then showcase student submissions at the beginning of the next class, highlighting the good work that has been created and modeling generous and supportive feedback. It is important that those discussions not focus on the student who made the work but instead on the work as a contribution shared with the class.

Our students often need support communicating with each other about technical ideas. Gaining this practice is one of the goals of the peer review days during which students bring completed drafts of their projects to class, pair (or triple) up, and work through a worksheet. The worksheet guides students to provide feedback on the work in front of them with a view towards the grading rubric. Not only does this help the student whose work is being evaluated, but it also gives the evaluator a chance to reflect on how others approached the same task, providing context for their own work, and also to reflect on how well they have personally satisfied the grading criteria. After sharing their written feedback, I ask the students to spend time orally discussing the feedback. In this way, students must talk technically and precisely about the feedback they wish to convey.

I also have students practice presentation skills in a larger classroom setting. On the day when students submit a project, we gather to complete a gallery walk. The format of the gallery will depend on the size of the artwork and the size of the class. In recent semesters, we have been able to gather around one table where all the artwork is displayed. (See Figure 5.) The students discuss their pieces, talking about some aspects of their inspiration, their process, characteristics of their work, and the emotions they had throughout the assignment. I like to think of this as a “victory lap” because everyone has learned something through the process and this is a milepost marking their achievements. I also install student work in an art gallery in a space on campus and hold a gallery opening as the culmination of the semester. Students can bring their friends to show off their work and everyone gets to feel like a true artist! (See Figure 6.)

Figure 5. Students engaging in a discussion about their artwork.

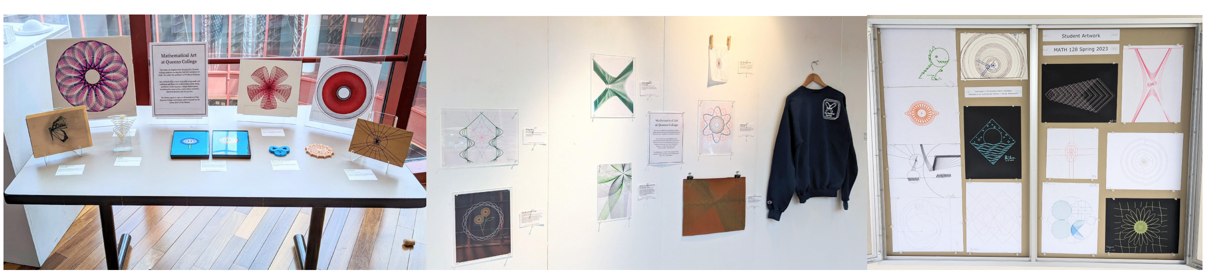

Figure 6. Student artwork displayed in a group exhibition and in a display case in the math department.

Grading Student Work

We have mentioned mathematical, artistic, coding, writing, and communication goals for *Mathematical Design*. The grading structure for the course and the projects attempts to holistically assess these goals. My approach aligns with Talbert's pillars of alternative grading (Talbert, 2021) in which student work is evaluated against clearly defined standards, student work is given helpful, actionable feedback, and students can revise and resubmit work without penalty.

We start by addressing the grading of the projects since that is the lion's share of a student's course grade. Mathematics instructors may be accustomed to single objectively correct answers. However, how does one grade obviously subjective artwork, code, and writing? I developed a transparent list of standards that reflect the goals of the course; here is an abridged summary of those criteria and what they each assess.

- **Engagement:** Are students making steady progress on their projects, attending in-class work days, and turning in their work on time?
- **Intentionality:** Is the artwork created through a deliberate design process that has been honed over time? Does the artwork match the artistic and mathematical terms discussed in the lab report?
- **Desmos Notebook Content:** Does the Desmos notebook use the mathematical and coding techniques taught in class?
- **Desmos Notebook Style:** Is the Desmos notebook organized into folders and with text cells that document what is happening?
- **Process Discussion:** Does the lab report discuss the process behind the piece, including inspiration, how the piece came to be, and evolved over time? What hurdles or successes were there?
- **Technical Discussion:** Does the lab report explain the mathematics and coding techniques you used with correct, technical language?
- **Artistic Discussion:** Does the lab report explain the artistic qualities you aimed for with correct, artistic language?
- **Writing style and format:** Is the lab report written in good English, flowing well from beginning to end, and following the formatting requirements?

Each criterion is graded on a scale of Exemplary (E) – Meets Expectations (M) – Revisions Needed (R) – Not Assessable (N) (Talbert, n.d.) and the distribution of scores corresponds to a grade on the classic A-B-C-D-F scale as follows:

- **A:** Earn a score of M or higher on all standards and a score of E on at least five standards.
- **B:** Earn a score of M or higher on all standards and a score of E on at least two standards.
- **C:** Earn a score of M or higher on six standards and no N scores.
- **D:** Earn a score of M or higher on five standards and at most one N score.
- **F:** Have fewer than five E or M scores OR earn two or more N scores.

Students have the opportunity to resubmit their work one time without penalty if so desired. I settled on such a grading scheme because I have an ideal about what each grade stands for. An A grade should correspond to excelling broadly across most aspects of the project, while a C grade says that a few aspects of the project are not up to expectations, even after having had the opportunity to revise and resubmit work. In my experience, this leads to a bimodal grade distribution with many A's and B's and many D's and F's, seemingly differentiating between people who put in the effort to meet the course expectations and those who don't.

Each project is graded independently and contributes a fixed percentage to the final grade, which is then rounded out by token contributions to a student's grade from homework and participation--daily homework submissions are graded for completion while in-class participation is graded based on presence and earnest contributions to class and group work.

The specific criteria and the rubric for assigning final grades themselves have been honed over time. They have been inspired by other classes I have taught as well as forays into standards-based grading. I don't claim to know the best way to grade, but I feel that the transparency provided by this system is making progress towards the ideal.

Student application of mathematical techniques

To provide further clarity about the mathematical and artistic concepts that students apply to create their artwork, let us explore these ideas in two student art pieces, shown in Figure 7.

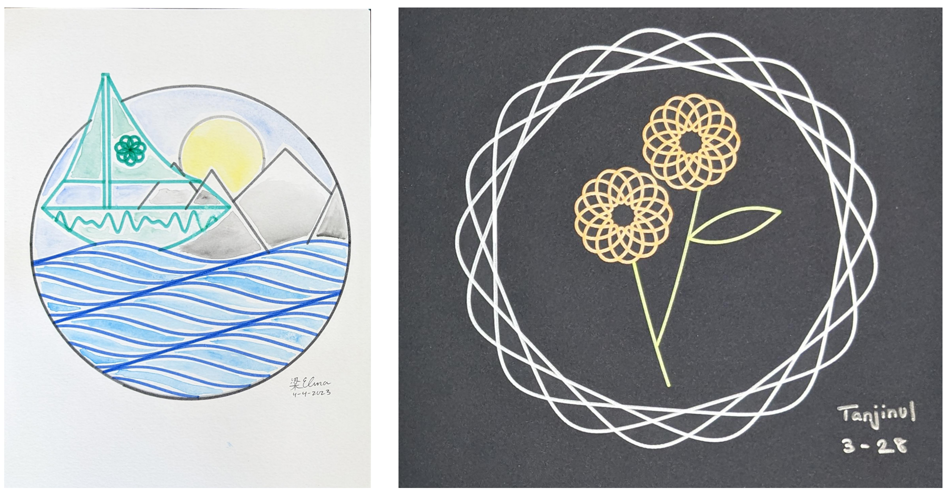

Figure 7. *Lakeside* by Elena Leung (left) and *Herb of the Sun* by Tanjinul Hoque (right).

In *Lakeside*, Elena Leung uses horizontal and vertical translations of a sine wave function and the design principle of repetition to create the waves of the water. She expertly employs function restrictions so that all the lines and curves stay within desired shape boundaries – for example, the reader can perceive that every water wave stops at the edge of the circle. She employs a variety of types of functions including linear functions for the mountains, quadratic functions for the sailboat, a tailored sum of trigonometric functions for the pattern on the side of the boat (all of which are standard functions) and circles for the boundary and sun types (an example of an implicit function). She also introduces an aspect of asymmetry by having the sails of the sailboat extend past the border of the circle. After using the plotter to create the scene on watercolor paper (requiring changing the pen multiple times), she completes the piece with watercolor paint.

In *Herb of the Sun*, Tanjinul Hoque evokes the shape of two marigolds, which have deep personal meaning for him. This scene features polar functions of the form $r = a \cdot cos(b \cdot sin(c \cdot \theta))$. He varies the parameters a, b, and c until the results match his desired aesthetic. Furthermore, these polar functions for the flower petals need to be translated from the center of the image to their final positions centered at (h, k), which requires converting a polar equation $r = f(\theta)$ into parametric equations of the form $x(t) = f(t) \cdot \cos(t) + h$ and $y(t) = f(t) \cdot \sin(t) + k$. The stems and leaves are domain-restricted quadratic functions. The impact of the image is strengthened by the use of multicolored metallic pens on a black background and generous negative space around the flowers.

Examples of Student Work

Figure 8 shows additional examples of student work from the past few years. There are examples of AxiDraw created artwork from Projects 2 and 3, and work on the 3D printer, the laser cutter, the embroidery machine, and even using a tattoo needle on fake skin from Project 3.

Figure 8. Examples of Student Work

Student Feedback

Many of the students who take this course report more positive experiences with the mathematics than they were expecting before the course. Below is a sampling of student quotes which show that *Mathematical Design* gave them more confidence in their mathematical and making abilities, helped them understand and appreciate interdisciplinary collaborations, and gave them the feeling that they belong to a community of makers.

> "*Mathematical Design* was like watching my best friend and sworn enemy become allies. I never realized that math and art could mix so well together. This also taught me how much

math is involved in art and vice versa. I am more comfortable with math, and not as hostile towards it. The Makerspace is now like my second home, and I go there whenever I have free time. Before everything was foreign and scary, I felt uncomfortable walking in. Now, I would like to continue expanding my experiences in that place and to learn how to use all the machines. I feel that the techniques I could learn there will really help my art skills and further my knowledge." – E.L., Spring 2023

"I feel like I have gained some valuable skills and grown a lot as an artist and maker this semester as well as a mathematician, especially since I ended the semester on such a high note due to my new skills with the embroidery machine, something I never thought I would be able to use in my lifetime. As an artist and creative, I get to embroider my art onto shirts to wear outside and be more open and proud about my creations, knowing that I made the art I am wearing. As a mathematician, I have learned many new functions and I can do more research, perhaps one day able to graph the fractals I so admire." – E.M., Spring 2023

"When it came time to select a math course in my senior year, I was dreading it. … All and all, this was the best math class I could've taken as an extremely visual learner. It was fun, engaging, and perfectly suited for my skill set as a designer and artist. I was able to expand my understanding of mathematics and art all in one course. Through every struggle and learning curve, I was still able to make incredible works of art that I didn't think were possible with mere math equations. For the first time in my education, I was glad to go to a math class, I didn't dread assignments, and I was able to understand what I was doing all while making fun and beautiful projects along the way. I am honored to have taken this course and learned so much." – H.R., Spring 2024

"The best surprise I received from doing this was being able to associate myself with others holding the same purpose as I did positively. There were a lot of people that worked in the QC Makerspace during the afternoon, with all sorts of them wanting to either make stuff for fun or finish assignments for their classes. … It was reassuring to see others going about their creative procedures differently than I was all at once, because I knew that if any of us were to ever need help, we could provide it to one another. … having [each] other's knowledge as a failsafe was extremely important. The alliances I made contributed greatly to how well my final product turned out to be, a pleasant surprise to behold." – J.G., Fall 2021

Instructor Takeaways

Making in a math course is very rare. It is a transformative process to hold a physical object in your hand that you conceived of in your brain and designed on the computer. For students to have an opportunity to experience this in a math course is special, and it helps to counter the prevailing narrative that mathematics is devoid of creativity (which mathematicians themselves would refute vigorously).

My favorite part about this course is how it empowers students to take ownership of their learning and grow as a mathematician, as a maker, and as an artist. At every turn, students are integrating their thoughts and lived experience with mathematical ideas. They bring their authentic selves to class when they share their inspirations, when they choose the topic of their artwork, and when they make choices along the way to create their final piece. I love giving students the agency to use their voice in their assignments and then amplifying their voices when we celebrate their work by putting their work in art exhibits.

The role that a supportive environment plays in enabling innovation in curriculum development cannot be overstated. I was supported by my department to create this course and pilot a section of the course with a small number of students. I was supported by the library when they provided a display case for student artwork to be displayed prominently and helped coordinate student access to resources. I was supported by the college Makerspace by providing a supervised space that students could use outside of class hours to experiment with and craft their artwork. I was supported by the NSF-funded faculty fellows program where I engaged with my cross-disciplinary colleagues who shared their experience developing “making” courses and best practices in their disciplines.

In iterating this course I have found a couple of aspects that lead to student success. First, my students have benefitted from having the projects extremely scaffolded, with a transparent timeline that has intermediate check-ins. For example:

- Gain Expertise by Week 2.
- Explore Possibilities by Week 3.
- Submit Complete draft for Peer Review by early in Week 5.
- Submit Final Project by the end of Week 5.

While I can rely on students to use the Makerspace outside of class, I have found it essential to have dedicated in-class work time; it allows me to provide guidance in level-appropriate ways (steering students away from projects that are too difficult or too easy), aide students who need help to make further progress, and see the intermediate progression of students to nudge students who are behind to put in the time. I also require students to submit two copies of their final artwork so that they can keep one and I can put one on display.

The process I through which I developed this course embodies the idea of productive failure I try to instill in my students. The current implementation of the course is stronger for this process. Whereas this course was imagined and developed before the COVID-19 pandemic that started in Spring 2020, this course piloted in Fall 2021, operated fully remotely, and using the Makerspace was out of the question. Instead, the course pivoted to the idea of making digital art, which honed our Desmos skills and taught us about online platforms for presentations and sharing work.

In Fall 2022, the campus reopened for in-person classes about halfway through the semester so the first half of the course was online and the second half was in person, and I adapted the course by having two projects: one piece of digital art, and a second piece using machines in the Makerspace. During that iteration I learned that my mathematical expectations were too ambitious and that I needed to give students more guidance about using the machines.

Classes in Spring 2023 and Spring 2024 were fully in person; however, students’ lives were still in flux with many demands outside of class in terms of work and caring for family members. In addition to dealing with changing online platforms (Campuswire, Discord, Microsoft Teams) in search of something that works for students, I developed Desmos tutorials and shared model homework submissions to provide additional guidance to build more support for student learning. And there is still more to improve in future iterations.

Innovation and productive failure in curriculum development is messy. If something is not working for most students, adapt to the reality and find a way to make the course and content meaningful to them. When I was a starting teacher I wanted to craft a learning environment where I was in full control of how everything happened in the classroom. As I have grown older, I have realized that it is best when I don’t have full control over how everything turns out to leave open the door to experimentation in the classroom and authentic student contributions. If anyone reading is inspired to implement similar changes in their pedagogy, realize that you can approach changes incrementally and that your students will appreciate the opportunity to be your partners on this journey.

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