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- ▶ Mathematics give precise answers; every answer is fallible.

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One or the other might make more sense depending on the situation.

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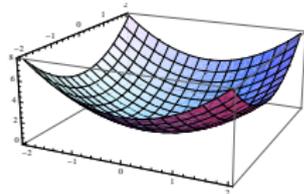
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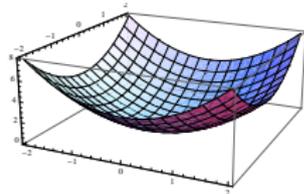
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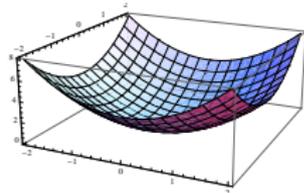
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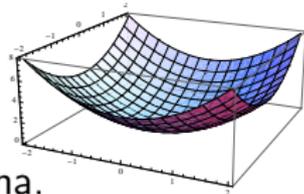
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Calculating minima of smooth functions: (You know how!)

- ▶ Differentiate with respect to each variable, and set equal to zero.
- ▶ Solve the resulting system of equations.
- ▶ Check to see if the solutions are local minima.



Least Squares Example

Example. Use the least-squares criterion to fit a line $y = mx + b$ to the data: $\{(1.0, 3.6), (2.1, 2.9), (3.5, 2.2), (4.0, 1.7)\}$.

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That is, the line that gives the least-squares fit for the data is

$$y = -0.605027x + 4.20332.$$

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- ▶ We'll learn how to use *Mathematica* to do this for us!

Price – Demand Curve (p. 111–114)

Example. A company is trying to determine how demand for a new product depends on its price and collect the following data:

price p	\$9	\$10	\$11
demand d	1200/mo.	1000/mo.	975/mo.

The company has reason to believe that price and demand are **inversely proportional**, that is, $d = \frac{c}{p}$ for some constant c .

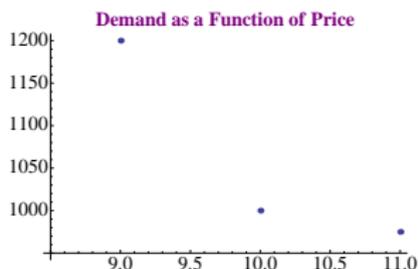
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→ Use the method of least squares to determine this constant c .



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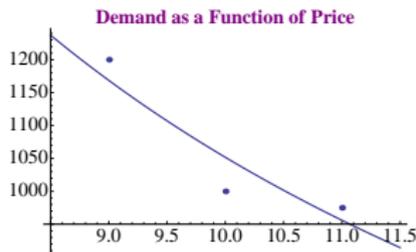
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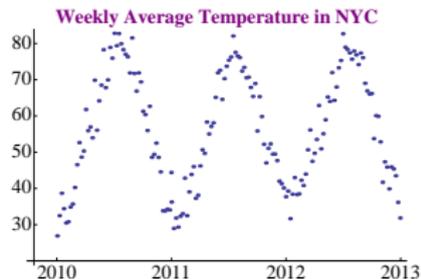
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Solving for c gives $c \approx 10517$.



New York City Temperature (similar to p. 158)

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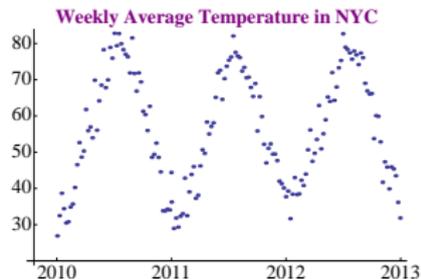


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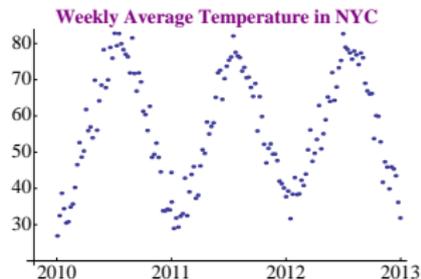
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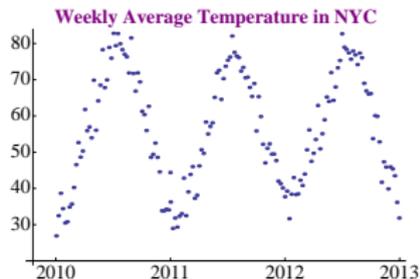
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Fitting to $Temp(t) = A + B \sin[2\pi(t - 0.3)]$ gives: $Temp(t) = 56.5 + 20.6 \sin[2\pi(t - 0.3)]$

