Modeling: Start to Finish

Example. Vehicular Stopping Distance

Background: In driver's training, you learn a rule for how far behind other cars you are supposed to stay.

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Formulation.

State the question. Identify factors. Describe mathematically.

Culminates with a mathematical model.

Mathematical Manipulation.

Determine mathematical conclusions.

Evaluation.

Translate into real-world conclusions. How good is the model?

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- ★ Does the two-second rule mean we'll stop in time?
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- v velocity
- t_r reaction time
- a vehicle acceleration / deceleration

Breaking down the problem

Describe mathematically.

Subproblem 1:	**Subproblem 2:**
Determine reaction distance d_r	Determine stopping distance d_b

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Therefore, the total stopping distance is $d_r + d_b = t_r v + Cv^2$.

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Reaction distance is tabulated in Table 2.4 and shown in Figure 2.14.

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- Examine methodology of data collection.
- Experimenters said $t_r = 3/4$ sec and calculated d_r!
- Perhaps we should design our own trial?

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Check fit: Plots observed stopping distance versus model. (Fig. 3.16)

- Model seems reasonable (through 70 mph).
- Residual plot shows additional behavior unmodeled (Fig. 3.17)

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- Suppose that **error in** is -10% for a true speed of $v = 60$. Then $v' = 54$ and the model predicts that stopping distance is $1.1 \cdot 54 + 0.054 \cdot 54^2 \approx 217$ instead of $1.1 \cdot 60 + 0.054 \cdot 60^2 \approx 260$.

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Is the model fruitful? Does it inspire other models?
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- This line of reasoning can be applied to any situation with constant deceleration.
- Come up with a good rule of thumb for drivers to follow and publicize it. (Next slide!)

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  - four seconds enough ($\leq$ 75 mph)
  - Add more if non-ideal road conditions.