1. (20 pts) (**QuAQ**'s) Quick answer questions.

   (a) We learned four types of errors in class, two of which might greatly affect your model. What are these two types, and why might they affect your model?

   (b) If A is a transition matrix and $\mathbf{x}_0$ is the initial distribution, what is a formula for the distribution at time 42? Explain briefly.

   (c) Is a Leslie matrix (that models population growth) a Markov chain matrix? Explain why or why not in one or two paragraphs.

   (d) Define an equilibrium distribution for a transition matrix A.

2. (15 pts) Let a and b be constant real numbers. Consider the linear program (LP):

$$\begin{array}{rr} \text{Maximize } ax + by & \\ \text{subject to} & 4y - x \leq 12 \\ & 2x - y \geq 4 \\ & x, y \geq 0 \end{array}$$

   (a) Draw the feasible region for (LP).

   (b) Depending on the constants a and b, (LP) can have different optimal solution(s). Describe all possible optimal solution(s).

   (c) Write the *Mathematica* command that will solve (LP) when $a = 3$ and $b = 2$. [*Important: DO NOT solve this linear program.*]

3. (10 pts) Suppose that in our waiting room simulation, the patients are clingy. In particular, suppose that as the patient is leaving the room, he turns around and says "Just one more thing, doctor..." and takes two more minutes of the doctor's time. (**Important:** This can happen multiple times in a row for the same patient.) Suppose that the probability that the patient asks for two more minutes upon leaving is 25%.

   (a) (3 pts) First, describe in words (one or two sentences) how this would affect the algorithm of the waiting room experience. You should describe what changes in the way the doctor sees her patients.

   (b) (7 pts) Now, explain how you would modify the provided code to take this into account. To receive partial credit, make sure to **explain in words** what you are trying to do in each line you write.

```
nwait = 0; busy = 0; endTime = 0;
For[i = 0, i < 180, i++,
   If[endTime == i, busy = 0];
   newPatient = If[RandomReal[] <= 0.075, 1, 0];
   If[newPatient == 1, nwait++];
   If[busy == 0 && nwait > 0, nwait--; busy = 1; endTime = i + 15];]
```

4. (5 pts) Consider the situation of the clingy patient from the previous problem. What is the **expected value** of the amount of **additional time** (time over 15 minutes) that the patient will request from the doctor?

   [*Hint: You may find it useful to recall that* $1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$ *for* $0 \leq x < 1$.]