

# Course Notes

Multivariable Calculus, Fall 2017

Queens College, Math 201

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<http://qc.edu/~chanusa/courses/201/17/>

# Class Introductions

Arrange yourselves into groups of four or five people,  
With people you **don't know**.

- ▶ Introduce yourself. (your name, where you're from, your major)
- ▶ What brought you to this class?
- ▶ Fill out **the blank side of** your notecard:
  - ▶ Write your name. (Stylize if you wish.)
  - ▶ Write some words about how I might remember you & your name.
  - ▶ *Draw* something (anything!) in the remaining space.
- ▶ *Small talk suggestion:* What kept you busy this summer?
- ▶ Exchange contact information. (phone / email / other)
- ▶ **Discuss!** What is *Calculus*?
  - ▶ Brainstorm! How do you convey *Calculus* to friends?
  - ▶ Organize into themes.
- ▶ How do these ideas translate to *multivariable* calculus?

## To do well in this class:

- ▶ **Form good study groups !!!!!!!!**
  - ▶ **Discuss** classwork and homework. Study for exams.
  - ▶ Bounce around ideas, topics, questions.
  - ▶ You will depend on this group.
- ▶ **Put in the time.**
  - ▶ Four credits = (at least) twelve hours / week out of class.
  - ▶ Homework stresses key concepts from class; learning takes time.
- ▶ **Come to class prepared.**
  - ▶ **Review** previous day's sections, notes.
  - ▶ **Do** the homework & prepare to present.
  - ▶ **Preview** the new day's sections. Download notes!
- ▶ **Stay in contact.**
  - ▶ If you are confused, ask questions (in class and out).
  - ▶ Don't fall behind in coursework or homework.
  - ▶ I need to understand your concerns.

All homeworks posted online; first one (many parts) due Wednesday.

# Homework policy:

There are two types of homework in this class:

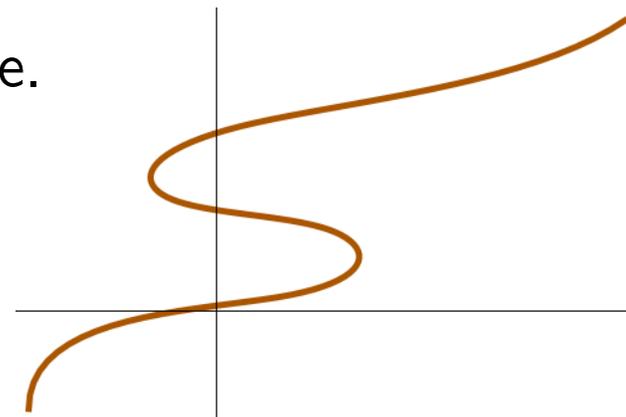
- ▶ **Daily:** Written / Presentation Homework.
  - ▶ A list of questions from the textbook to practice.
  - ▶ If a question is hard, you should practice **more** like it.
  - ▶ Presentations at beginning of the next class.
    - ▶ Write up solution in bullet-point format.
    - ▶ Present the solution to the class & answer questions.
  - ▶ One of only two bonus point opportunities in this class.
  - ▶ Starts Wednesday August 30! (+ Blackboard quiz)
- ▶ **Weekly:** Online Homework.
  - ▶ Using online homework called **Webwork**.
    - ▶ Link on webpage to:  
<http://192.195.176.176/webwork2/QC201/>
    - ▶ Your username: QC email username. (like **Christopher.Hanusa55**)
    - ▶ Initial password: CUNYFirst ID # (like **10111213**)
  - ▶ First assignment due Wednesday September 6. (13 Qs)
  - ▶ \* Get started early! \*

# Parametric Curves

Imagine a particle traveling along this curve.

Is the curve a function? ( $y = f(x)$ ?)

As an alternative, we can write the  $x$ -coordinate and the  $y$ -coordinate of the particle as a function of “time”.



(Write  $x = f(t)$  and  $y = g(t)$ .)

This pair of functions is called the **parametric equations** of the curve.

And the variable  $t$  is called a **parameter**.

*Note:* The **domain** of  $t$  is often  $(-\infty, \infty)$  or an interval  $a \leq t \leq b$ .

**Goal 1:** *Understand* parametric curves. (Today)

**Goal 2:** *Do calculus* using parametric curves. (Next time)

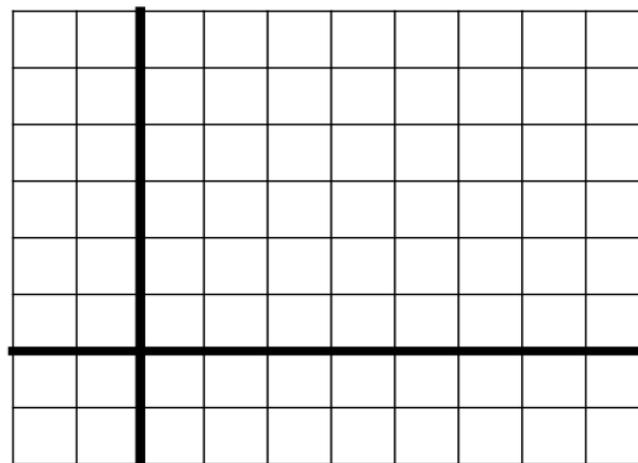
# Sketching Parametric Curves

What is the shape of a curve given by parametric equations?

- ▶ By hand ← How do you plot  $y = f(x)$ ?
- ▶ Use a calculator or computer

**Example.** Plot the curve defined by  $x(t) = t^2 - 2t$  and  $y(t) = t + 1$ .

$t$	-1	0	1	2	3	4
$x(t)$						
$y(t)$						



The shape of the curve is \_\_\_\_\_.

Should we have known this?

**Key concept:** Eliminate the parameter  $t$  to combine  $x = f(t)$  and  $y = g(t)$  into a “normal” function  $y = F(x)$  or  $x = F(y)$ .

Solve for  $t$  in second equation:  $t = y - 1$  and plug in:

$x = (y - 1)^2 - 2(y - 1) = y^2 - 4y + 3$ , a “sideways parabola”.

# Around and Around

**Example.** Plot the curve defined by  $x = \cos t$ ,  $y = \sin t$ ,  $0 \leq t \leq 2\pi$ .

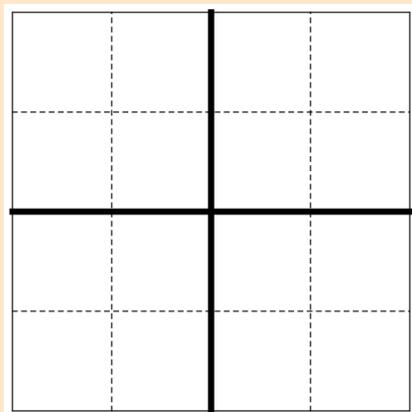
Plot points or **solve directly**.

$$y = \sin(\cos^{-1} x)$$

$$y = \sqrt{1 - x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$

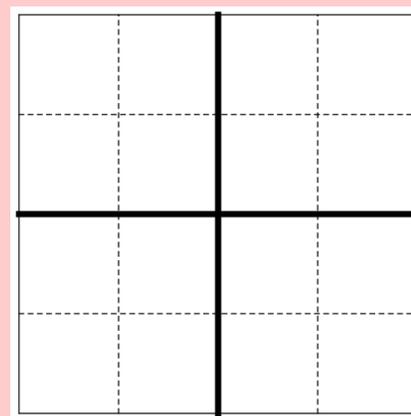


A circle! But you knew that.

$$\cos^2 t + \sin^2 t = 1$$

Starts at  $t = 0$ :  $(1, 0)$  and goes around counterclockwise.

**Example.** Is this the same as  $x = \cos 2t$ ,  $y = \sin 2t$ ,  $0 \leq t \leq 2\pi$ ?



**Question:** What is  $x^2 + y^2$ ?

The figures traced out (the curves) are the same but the functions **are not** the same.

You need to know your trig functions and values at certain points!!!!

## Circumnavigation

If we want to draw a circle at some other place

$$(x - h)^2 + (y - k)^2 = r^2,$$

set  $x - h = r \cos t$  and  $y - k = r \sin t$ .

In other words, use the parametric equations

$$x(t) = r \cos t + h \quad \text{and} \quad y(t) = r \sin t + k.$$

**Try it out!** Get out your graphing calculator TI-( $\leq 86$ ).

Switch to Parametric mode: `MODE` ↓ ↓ ↓ `PAR` (`Enter`).

Set the domain of  $T$  to be from 0 to  $2\pi$ .

`WINDOW`: `Tmin` = 0, `Tmax` =  $2\pi$ , `Tstep` =  $\pi/10$ .

Enter the equations  $X_1 = 3 \cos(T) + 2$  and  $Y_1 = 3 \sin(T) + 4$ .

This plots a circle of radius 3 centered at (2, 4).

## Computers to the rescue

Calculators and computers can graph much more complicated curves.

$$x_1(t) = t + 2 \sin(2t) \quad \text{and} \quad y_1(t) = t + 2 \cos(5t)$$

$$x_2(t) = 1.5 \cos t - \cos 30t \quad \text{and} \quad y_2(t) = 1.5 \sin t - \sin 30t$$

$$x_3(t) = \sin(t + \cos 100t) \quad \text{and} \quad y_3(t) = \cos(t + \sin 100t)$$

### Tools:

- ▶ Wolfram Alpha <http://www.wolframalpha.com/>
- ▶ More powerful is Wolfram *Mathematica*. Get license from MyQC: [myqc.qc.cuny.edu/Academics/mathematics/Pages3/access.aspx](http://myqc.qc.cuny.edu/Academics/mathematics/Pages3/access.aspx)
- ▶ Online plotter: [desmos.com](https://www.desmos.com) Put  $(f(t), g(t))$  in parentheses.  
<https://www.desmos.com/calculator/ndgy5rppqh>

**Next time:** What is the shape of a parametric curve? What is the length of a parametric curve? What about polar coordinates?

**Before then:** Work on homework to present in class Wednesday. Email me contact info, do syllabus quiz. Play with parametric eqns.